Hamiltonian mechanics and the semiclassical quantum mechanics

These lectures continue the course "Topology and Physics" taught by H. Postuma and M. Vonk. The goal is to introduce in greater then usual details Hamiltonian mechanics, some basic notions of symplectic geometry and the semiclassical quantum mechanics. There will be more geometry than topology, however, an important part of the semiclassical analysis, the Maslov index, is a topological phenomenon.

Tentative schedule by lectures.

(1) Lagrangian mechanics a manifold. Legendre transform. Hamiltonian mechanics on a cotangent bundles.
(2) Hamilton-Jacobi theory and the variational principle. Boundary conditions and Lagrangian fibrations.
(3) Basic facts in quantum mechanics. Quantization of the phase space and operators in Hilbert spaces. Time evolution operator. Schodinger equation.
(4) Path integral for the time evolution operator in quantum mechanics. Integration over paths in the configuration space with Dirichlet boundary conditions. Relation to the operator formulation in quantum mechanics.
(5) Path integral representation of the evolution operator in the phase space with boundary data given by Lagrangian fibrations. Relation to operators in the Hilbert space.
(6) Making the path integral mathematically meaningful. Path integral as a formal semiclassical theory. Maslov index. Theorem: the formal semiclassical path integral satisfies the Schodinger equation.

Prerequisits. If you are continuing with the course "Topology and Physics", you have all prerequisites. Otherwise a basic prior knowledge of manifolds, symplectic geometry and quantum mechanics is useful. On the other hand I am planning to introduce basics of symplectic geometry and basics of quantum mechanics here.

Lecture notes will be available. This course will be taught by zoom and lectures will be recorded.