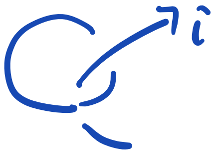
Final Exam

1) Let $\mathcal{C}$ be a modular tensor category. Compute invariants of colored links:

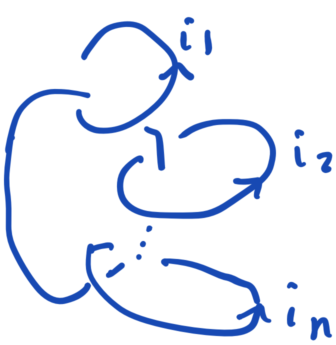
(i)  here and below an uncolored, unoriented connected component of a link means Kirby color:

$$\Big| = \sum_i d_i \uparrow^{i} = \sum_i d_i \downarrow^{i} \quad (d_i = d_{i^*})$$

(ii) 

(iii) 

K

(iv) 

2) Let $\mathcal{C}_q$ be the modular tensor category which is the quotient category of $U_q(sl_2)$-modules. In this category simple objects V_i are enumerated by $i = 0, 1, \ldots, r-2$,

$q = e^{\frac{i\pi m}{r}}$, $\quad d_k = \dfrac{q^{k+1} - q^{-k-1}}{q - q^{-1}}$. (Lect 11)

r - odd, m & r are mutually prime

$1 \le m \le r-1$

(i) Prove that for $m = 1$ all $d_k > 0$

(ii) Prove that if $m = 2$ there are $d_k < 0$

(iii) Prove this for $m = 3$

(iv) Is it possible that $d_k = 0$ for some k (for some $m > 1$)?

3) In this category $\mathcal{C}_q$

(i) compute $\mathcal{D} = \sum_{k=0}^{r-2} d_k^2$

(ii) For $q = e^{\frac{i\pi}{r}}$ compute $p_\pm = \sum_{k=0}^{r-2} v_k^{\pm 1} d_k$

4) Let $\tilde{\mathcal{C}}_q$ be the monoidal tensor category related to $\mathbb{Z}_n$ (see Lect. 11).

(i) Compute D

(ii) Compute $p_\pm$

(iii) For a framed link L with m connected components $L_1, .., L_m$ colored by simple objects $V_{j_1}, .., V_{j_m}$ prove that

$$inv(L_{j_1..j_m}) = q^{\sum_{\alpha\beta} j_\alpha L_{\alpha\beta} j_\beta}$$

Here $L_{\alpha\beta}$ is the linking number between components L_α and L_β, $L_{\alpha\alpha}$ is the number of twist of the framing along component L_α. $(L_{\alpha\beta})$ is called the linking matrix of a framed link L

(iv) Compute the invariant of M_L obtained from S_3 by surgery along a knot L with trivial framing.