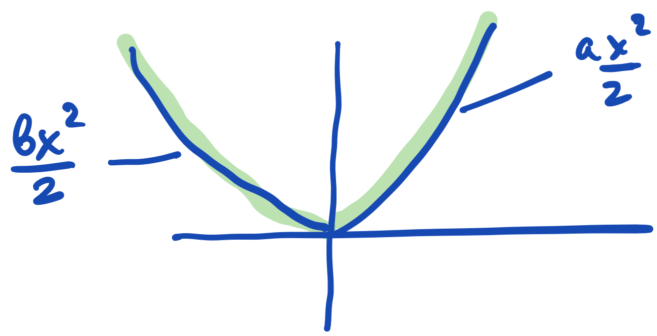
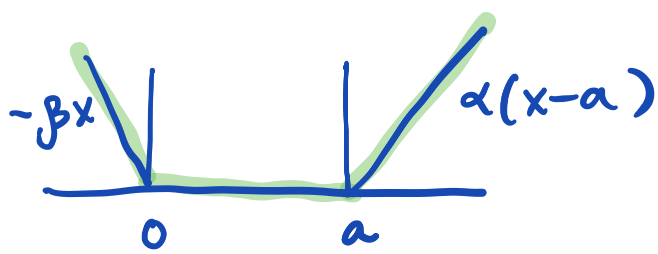
Homework 1

1. (a) Are boundary conditions $\vec{q}(t_1) = \vec{Q}$, $\dot{\vec{q}}(t_2) = \vec{V}$ variational?

   (b) Apply this to $\mathcal{L}(\xi, q) = \frac{m(\xi, \xi)}{2}$ and find corresponding solution to Euler-Lagrange equations

2. Find the Legendre transform of

   (a) 

   (b) 

3. Does the bracket

$$\{F, G\} = g(x,y)\,\frac{\partial F}{\partial x}\frac{\partial G}{\partial y}$$

on $C^\infty(\mathbb{R}^2)$ satisfy Jacobi identity?

Here $g(x,y)$ is a smooth function.

4. Let $\mathfrak{g}$ be a finite dimensional Lie algebra and $\{e^i\}$ be a basis. It defines structural constants:

$$[e_i, e_j] = \sum_k c_{ij}^k e_k$$

Define the brackets

$$\{F, G\} = \sum_{ij,k} c_{ij}^k \, x_k \frac{\partial F}{\partial x_i} \frac{\partial G}{\partial x_j}$$

on $C^\infty(\mathfrak{g}^*)$. Here $\{x_i\}$ are coordinates on $\mathfrak{g}^*$ corresponding to the basis $\{e^i\}$ in $\mathfrak{g}^*$, which is dual to $\{e_i\}$ in $\mathfrak{g}$.

Prove that this is a Poisson structure on $C^\infty(\mathfrak{g}^*)$.