HW 4

1) Quantum system with the algebra of observables $A = \mathrm{End}(\mathbb{C}^N)$ is in a state with the density matrix $\hat{\rho}$. Let a be an observable with eigenvalues $\{a_i\}$. What is the probability that in the state $\ell_{\hat{\rho}}(\cdot) = tr(\hat{\rho}\,\cdot)$ the observable a has value a_i.

2) Finish the proof of the inequality

$$\Delta_{\hat{\rho}}(a)^2\, \Delta_{\hat{\rho}}(b)^2 \geqslant \frac{1}{4}\, \Delta_{\hat{\rho}}(i[a,b])^2$$

where $\Delta_{\hat{\rho}}(a)^2 = tr(\hat{\rho}(a - tr(\hat{\rho}a))^2)$

$\hat{\rho}^* = \hat{\rho}$, $a^* = a$, $b^* = b$ are Hermitian operators in a Hilbert space.

3) Prove the identity for $N \times N$ matrices

$$e^B A e^{-B} = \sum_{n=0}^{\infty} \frac{1}{n!} \underbrace{[B[B \cdots [B,A] \cdot\cdot]}_{N}$$

4) Prove that in a Poisson algebra the power series (assume the convergence)

$$a(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \{H, \{H, \ldots, \{H, a\}\ldots\}$$

solves the differential equation

$$\frac{da(t)}{dt} = \{H, a(t)\}$$

5) For quantum mechanics of a point particle in $\mathbb{R}^N$ find quantum states which in the limit $\hbar \to 0$ converge to the classical probabilistic state

$$\ell_\rho : C^\infty(\mathbb{R}^{2N}) \longrightarrow \mathbb{R}$$

$$\ell_\rho(A) = \int_{\mathbb{R}^{2N}} A(p,q)\,\rho(p,q)\, d^N p\, d^N q$$

where $\int_{\mathbb{R}^{2N}} \rho(p,q)\, d^N p\, d^N q = 1$, $\rho(p,q) \geq 0$.