

HW6

1) Prove Wick's formula

2) Prove:

$$1 + \sum_{\Gamma} (ih)^{-\chi(\Gamma)} \frac{F(\Gamma)}{|\text{Aut}(\Gamma)|} = \exp\left(\sum_{\Gamma \text{ connected}} (ih)^{-\chi(\Gamma)} \frac{F(\Gamma)}{|\text{Aut}(\Gamma)|} \right)$$

3) Prove that

$$G_0(\tau, \tau') = \begin{cases} \tau \frac{t - \tau'}{t}, & \tau < \tau' \\ \tau' \frac{t - \tau}{t}, & \tau > \tau' \end{cases}$$

is the unique solution to

$$-\frac{d^2}{dt^2} G_0(\tau, \tau') = \delta(\tau, \tau')$$

$$G(0, \tau') = G(t, \tau'), \quad G(\tau, \tau') = G(\tau', \tau)$$

4) Prove that

$$G(\tau, \tau') = G_0(\tau, \tau') + O(|\tau - \tau'|)$$

as $\tau \rightarrow \tau'$.